

**Class notes, problem sets, problem set solutions (your own and those distributed in class), and a calculator may be used. No textbooks or other materials are allowed.**

**ATTEMPT ALL PROBLEMS AND SHOW ALL WORK! This exam has a maximum score of 100 points. You have 80 minutes to complete this exam. Your exam should be written in a bound examination blue book.**

1. Short-answer problems. Please answer concisely but clearly.

- (a) [10 points] Explain why the energy of the lowest confined state in a finite-barrier electron quantum well is lower than that of the lowest confined state in an electron quantum well with infinitely high barriers.
- (b) [10 points] Explain why, in the conversion of the strain, stress, elastic stiffness and elastic compliance tensors from tensor to matrix notation, factors of 2 or 4 must be introduced in  $\varepsilon_\alpha$  and  $s_{\alpha\beta}$ . What numerical factor  $K$  would need to be introduced into the relation  $s_{\alpha\beta} = Ks_{ijkl}$  for  $s_{14}$ ,  $s_{15}$ , and  $s_{44}$  ( $K$  may be different for each of these)? Explain.

- (c) [10 points] Consider the displacement of a 2D sheet described by a displacement tensor

$$e_{ij} = \begin{pmatrix} 0.04 & -0.02 \\ 0.02 & 0.03 \end{pmatrix}.$$

Compute the resulting strain tensor  $\varepsilon_{ij}$ , the fractional change in area, and the angle of rotation (with units).

- (d) [10 points] Explain why semiconductor heterojunction lattice mismatch can be accommodated only by edge, and not screw, dislocations.

2. [30 points] Consider a semiconductor that is characterized by the following relationship between  $E$  and  $\mathbf{k}$  in the conduction band, where  $E_c$  is the energy of the conduction-band edge:

$$E(\mathbf{k}) = \sqrt{E_c^2 + E_c \frac{\hbar^2 k^2}{m^*}}$$

- (a) Suppose this semiconductor is used to form a quantum well of width  $L$  with infinite barriers. Calculate the density of states  $g_{2D}(E)$  for the lowest quantum-confined state.
- (b) Show that, for sufficiently small values of  $(E-E_c)$ ,  $g_{2D}(E)$  calculated in part (a) is approximately equal to the density of states obtained for a simple parabolic band structure with effective mass  $m^*$ , i.e.,  $g_{2D}(E) = m^* / \pi \hbar^2$ .
- (a) Discuss the correspondence of the 2D density of states from part (a) with the 3D bulk density of states that would arise from the same energy band structure, assuming a quantum well of width  $L$  with infinite barriers.

3. [30 points] Consider a heterojunction that is formed between two semiconductor materials, A and B, with crystal structures that possess hexagonal symmetry, e.g., the wurtzite crystal structure. In such crystals (as well as in cubic crystals for certain geometries), a sheet of charge can form at the surfaces of or interfaces formed by each material due to the presence of strain; this is called the piezoelectric effect.

For an interface formed between A and B within the  $xy$ -plane, suppose the sheet charge density at the interface arising from this effect is given by

$$\sigma = \begin{cases} +P_A \varepsilon_{\parallel} \\ -P_B \varepsilon_{\parallel} \end{cases} \text{ for } \begin{cases} \text{semiconductor A} \\ \text{semiconductor B} \end{cases}.$$

For simplicity we assume the elastic stiffness constants for the two materials are the same. The unstrained lattice constants in the  $xy$ -plane are  $a_A$  and  $a_B$  for materials A and B, respectively.

- (a) Obtain an expression for  $a_{\parallel}$  in equilibrium assuming the heterojunction is coherently strained, i.e., there are no dislocations present, if the thickness of material A is twice that of material B. Your result should be in terms of  $a_A$  and  $a_B$ .
- (b) Using the expression given above, compute the sheet charge densities in each material at the interface arising from strain:  $\sigma_A$  in semiconductor A and  $\sigma_B$  in semiconductor B. Such charges play an essential role in determining the electronic properties of heterostructures in wurtzite semiconductors such as GaN and ZnO.