Problem Set 4 **ECE 236A** Due February 28, 2008
IN CLASS

Problem solutions are to be turned in at the beginning of class on the due date. Solutions to all problems will be provided after problem sets are collected in class. SHOW ALL WORK.

1. In class we derived the density of states for a two-dimensional quantum-well structure. Now consider a one-dimensional quantum "wire" structure with infinite barriers leading to confinement in the *y*- and *z*-directions, i.e., the wire axis is along the *x*-direction. The wire dimensions in the *y*- and *z*-directions are L_y and L_z, respectively, with $L_y<L_z$. The envelope functions for electrons may be assumed to be periodic in the *x*-direction with period *L*. Assuming the bulk energy band dispersion relation given below, obtain an expression for the electronic density of states in the quantum wire for the lowest-energy confined state.

$$E(\mathbf{k}) = E_c + \frac{\hbar^2 k^2}{2m^*}$$

2. Show that the eigenvalues of the Luttinger Hamiltonian H_L, discussed in class and given below, will yield hole effective masses $m^* = m_e/(\gamma_1 \pm 2\gamma_2)$, where m_e is the free electron mass, for **k** in the (100), (010), and (001) directions in reciprocal space. Work this out by hand, not using a symbolic math program. The components of the operator $\hat{\mathbf{J}}$ were given in class.

$$H_L = -\frac{\hbar^2}{2m_e}\left[\left(\gamma_1 + \tfrac{5}{2}\gamma_2\right)k^2 - 2\gamma_3\left(\hat{\mathbf{J}}\cdot\mathbf{k}\right)^2 + 2\left(\gamma_3 - \gamma_2\right)\left(\hat{J}_x^2 k_x^2 + \hat{J}_y^2 k_y^2 + \hat{J}_z^2 k_z^2\right)\right]$$

3. Consider an n-type modulation-doped heterostructure in which a two-dimensional electron accumulation layer, or two-dimensional electron gas, is formed in GaAs at the interface between GaAs and a barrier semiconductor material. Compute and plot the electron sheet concentration n_s as a function of ΔE_c for ΔE_c = 0.15eV to 0.5eV. In computing the energies of the confined states, you may assume that the quantum well is triangular and the barrier infinite in height, and that the Fermi distribution function can be approximated by its form at zero temperature. Assume the following values in your computation: $m^*=0.067m_e$; $\varepsilon=12.9\varepsilon_0$ for both the barrier and GaAs layers; $N_d=1\times10^{18}\text{cm}^{-3}$ in the barrier; $E_c-E_F=0.1\text{eV}$ in the barrier outside the depletion region.