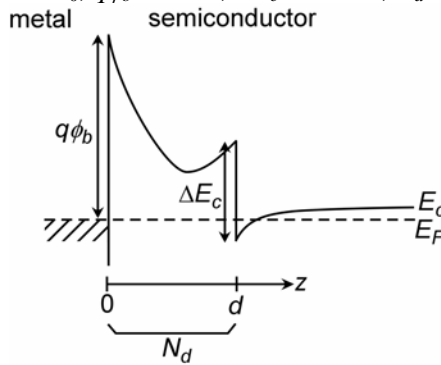


**Problem solutions are to be turned in at the beginning of class on the due date. Solutions to all problems will be provided after problem sets are collected in class. SHOW ALL WORK.**

1. Consider a two-dimensional electron gas structure as shown below, in which the width of the barrier is finite, and the barrier is assumed to be fully depleted. Assume that the effective mass  $m^*$  and dielectric constant  $\epsilon$  are the same for both semiconductor materials in the heterostructure, and that the Fermi distribution function may be approximated as a step function.

- (a) Obtain an implicit equation for the equilibrium electron sheet concentration  $n_s$  in the two-dimensional electron gas using an approach analogous to that used in class. Your result should be in terms of  $m^*$ ,  $\epsilon$ ,  $a_1$  (magnitude of the first root of the Airy function) and the parameters shown in the figure below. You may assume that only one subband in the quantum well is occupied.
- (b) Now assume a bias voltage  $V$  is applied to the metal contact at  $z=0$ . Obtain an implicit equation for  $n_s$  as a function of  $V$  for  $V < 0$ . Plot (giving numerical values)  $n_s$  vs.  $V$  for  $-1V < V < 0$  assuming  $m^* = 0.067m_0$ ,  $\epsilon = 13\epsilon_0$ ,  $q\phi_b = 1.0\text{eV}$ ,  $\Delta E_c = 0.3\text{eV}$ ,  $N_d = 2 \times 10^{18}\text{cm}^{-3}$ , and  $d = 30\text{nm}$ .



2. Consider a semiconductor with the following (simple) structure for the conduction and valence bands:

$$E_c = E_g + \frac{\hbar^2 k^2}{2m_e^*} \quad E_v = -\frac{\hbar^2 k^2}{2m_h^*}$$

- (a) Compute the joint density of states for this semiconductor, in bulk material.
  - (b) Compute the joint density of states for this semiconductor, assuming it is used to form a quantum well of width  $L$  with infinite barriers. Consider only the  $n=1$  confined electron and hole states.
3. Show that  $\langle \psi_2 | \mathbf{p} | \psi_1 \rangle = (im/\hbar)(E_2 - E_1)\langle \psi_2 | \mathbf{r} | \psi_1 \rangle$ , where  $\psi_1$  and  $\psi_2$  are eigenfunctions of the Hamiltonian  $H_0 = p^2/2m + V(\mathbf{r})$ . Hint: compute the commutator  $[\mathbf{r}, H_0]$  and its matrix element.